

JEE Main Home Practice Test - 2 | JEE - 2024

Date: 04/01/2024

Maximum Marks: 300

Timing: 10:00 AM to 1:00 PM

Duration : 3.0 Hours

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300**.
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2** contains **10 Numerical Value Type Questions** Out of which **ONLY 5 (any)** questions have to be attempted. You will **NOT** be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.
The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
6. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

Syllabus

Physics: Full Syllabus

Chemistry: Full Syllabus

Mathematics: Full Syllabus

Name of the Candidate (In CAPITALS) :

Roll Number :

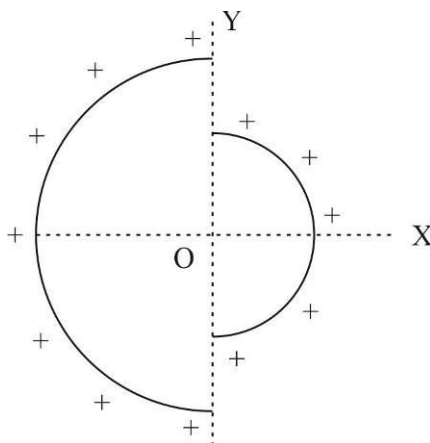
OMR Bar Code Number :

Candidate's Signature : Invigilator's Signature

PART - I : PHYSICS**100 MARKS****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. Two semicircular wires of radius 20cm and 10cm have a common-centre at the origin O as shown in the figure. Assume that both the wires are uniformly charged and have an equal charge of 0.70nC each. The magnitude of electric field at the common centre of curvature O of the system is:

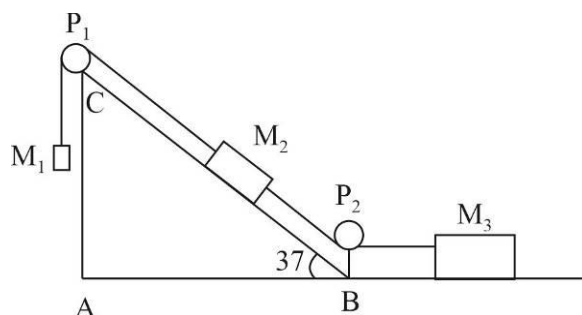


- (A) 100 V/m (B) 301 V/m (C) 401 V/m (D) 501 V/m
2. A pendulum suspended from the roof of an elevator at rest has a time period T_1 , when the elevator moves up with an acceleration a its time period becomes T_2 , when the elevator moves down with an acceleration a , its time period becomes T_3 , then:

- (A) $T_1 = \sqrt{T_2 T_3}$ (B) $T_1 = \sqrt{T_2^2 + T_3^2}$
- (C) $T_1 = \frac{T_2 T_3 \sqrt{2}}{\sqrt{T_2^2 + T_3^2}}$ (D) $T_1 = \frac{T_2 T_3}{\sqrt{T_2^2 + T_3^2}}$

SPACE FOR ROUGH WORK

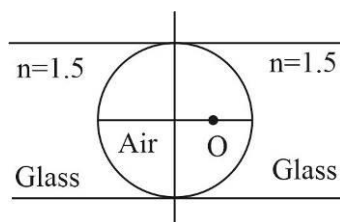
3. Two identical bodies M_2 and M_3 each of 4kg are tied to a massless inextensible string which is made to pass around pulleys P_1 and P_2 as shown in figure. Angle $ABC = 37^\circ$. The coefficient of kinetic friction between the bodies and the surface on which they slide is 0.25. If the body M_1 moves down with uniform speed, find the value of mass M_1 .



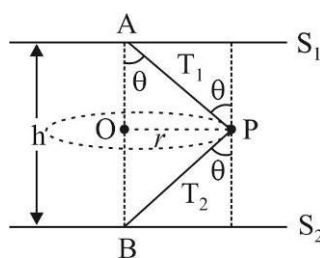
- (A) 36.8 kg (B) 9.8 kg (C) 4.2 kg (D) 2.1 kg
4. A vertical cylinder closed from both ends is equipped with an easily moving piston dividing volume into two parts, each containing one mole of air. In equilibrium at $T_0 = 300K$ the volume of the upper part is $\eta = 4.0$ times greater than that of the lower part. At what temperature will the ratio of these volumes be equal to $\eta' = 3.0$?
- (A) 329.1K (B) 403.2K (C) 239.1K (D) 421.9K
5. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as:
- (A) $r_n \propto \sqrt{n}$, $E_n \propto n$ (B) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$
- (C) $r_n \propto n$, $E_n \propto n$ (D) $r_n \propto \sqrt{n}$, $E_n \propto \frac{1}{n}$

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6. Two concave refracting surfaces of equal radii of curvature R face each other in air as shown in figure. A point object O is placed midway between the centre and one of the poles. Then, the separation between the images of O formed by each refracting surfaces is:



- (A) $11.4 R$ (B) $1.14 R$ (C) $0.12 R$ (D) $0.0114 R$
7. Two identical strings with fixed ends separated by height h have their other ends tied to body P of mass m as shown in figure. When the body rotates with uniform angular speed $2\sqrt{2g/h}$ in a horizontal plane about the vertical axis, the ratio of tensions (T_1/T_2) in the string is:



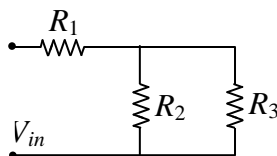
- (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) $\frac{2}{5}$ (D) $\frac{5}{2}$
8. If the Broglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency $6 \times 10^{14} \text{ Hz}$, then the speed of electron is equal to :

(Speed of light $= 3 \times 10^8 \text{ m/s}$ Planck's constant $= 6.63 \times 10^{-34} \text{ J.s}$ Mass of electron $= 9.1 \times 10^{-31} \text{ kg}$)

- (A) $1.8 \times 10^6 \text{ m/s}$ (B) $1.1 \times 10^6 \text{ m/s}$ (C) $1.45 \times 10^6 \text{ m/s}$ (D) $1.7 \times 10^6 \text{ m/s}$

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9. The temperature at which average de-Broglie wavelength of helium atom becomes 0.5 nm is:
 (A) 6.6 K (B) 7.1 K (C) 279.6 K (D) 280.1 K
10. An electron orbiting around a nucleus has angular momentum L . The magnetic field produced by the electron at the centre of the orbit can be expressed as:
 (A) $B = (\mu_0 e / 8\pi m r^3) L$ (B) $B = (\mu_0 e / 4\pi m r^3) L$
 (C) $B = (\mu_0 e / 6\pi m r^3) L$ (D) $B = (e / 4\pi \epsilon_0 m r^3) L$
11. Two bodies of mass M_1 and M_2 are kept separated by a distance d . The potential at the point where the gravitational field produced by them is zero, is:
 (A) $-\frac{G}{d} (M_1 + M_2 + 2\sqrt{M_1 M_2})$ (B) $-\frac{G}{d} (M_1 M_2 + 2\sqrt{M_1 + M_2})$
 (C) $-\frac{G}{d} (M_1 - M_2 + 2\sqrt{M_1 M_2})$ (D) $-\frac{G}{d} (M_1 M_2 - 2\sqrt{M_1 + M_2})$
12. For ensuring dissipation of same energy in all three resistors (R_1, R_2, R_3) connected as shown in figure, their values must be related as:

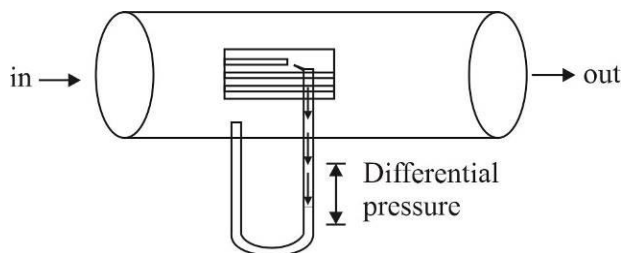


- (A) $R_1 = R_2 = R_3$ (B) $R_2 = R_3$ and $R_1 = 4R_2$
 (C) $R_2 = R_3$ and $R_1 = \frac{1}{4} R_2$ (D) $R_1 = R_2 + R_3$
13. A thin ring has a radius R , density ρ and Young's modulus Y . The ring is rotated in its own plane about an axis passing through its centre with angular velocity ω . Then, the small increase in its radius is:

(A) $dR = \frac{\rho \omega^2 R^3}{Y}$ (B) $dR = \frac{3\rho \omega^2 R^3}{Y}$ (C) $dR = \frac{6\rho \omega^2 R^3}{Y}$ (D) $dR = \frac{\rho \omega^2 R^3}{2Y}$

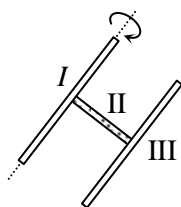
SPACE FOR ROUGH WORK

14. The Pitot tube shown in the figure is used to measure fluid flow velocity in a pipe of cross-sectional area S . It was invented by a French engineer Henri Pitot in the early 18th century. The volume of the gas flowing across the section of the pipe per unit time is:
(The difference in the liquid columns is Δh , ρ_0 and ρ are the densities of liquid and the gas respectively)



- (A) $Q = 2S \sqrt{\frac{\Delta h \rho_0 g}{\rho}}$ (B) $Q = S \sqrt{\frac{2\Delta h \rho_0 g}{\rho}}$
(C) $Q = S \sqrt{\frac{\Delta h g \rho_0}{\rho}}$ (D) $Q = S \sqrt{\frac{2\Delta h g \rho}{\rho_0}}$

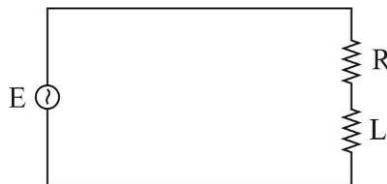
15. A structure in the shape of letter H is formed with the help of three identical rods each of length l . The system can rotate along axis I . The angular speed of the system when plane of H becomes vertical from its original position of rest along the horizontal is:



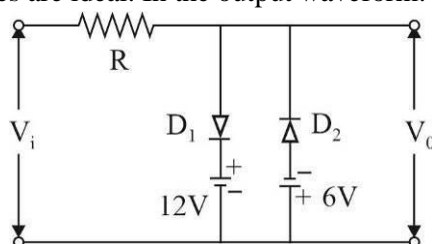
- (A) $\frac{3}{2} \sqrt{\frac{g}{l}}$ (B) $\frac{2}{3} \sqrt{\frac{g}{l}}$ (C) $\frac{1}{3} \sqrt{\frac{g}{l}}$ (D) $\frac{3}{4} \sqrt{\frac{g}{l}}$

SPACE FOR ROUGH WORK

16. The impedance of the R - L circuit given in the adjacent figure is expressed by the relation $Z^2 = A^2 + B^2$. Then, the dimensions of AB are:



- (A) $[M^1 L^2 I^{-2} T^{-3}]$ (B) $[M^2 L^4 I^{-4} T^{-6}]$ (C) $[M^1 L^{-1} I^{-2} T^{-3}]$ (D) $[M^{-1} L^{-2} I^2 T^{-4}]$
17. A LASER source of heat of power $1.2W$ is placed very close to one end of a rod of cross-section area $3cm^2$ and thermal conductivity $400W/mK$. The length of the rod (L) required to maintain a temperature difference of $10^\circ C$ across its ends is: (Assume that all power emitted by the source falls on the rod)
- (A) $1.5 m$ (B) $2.2 m$ (C) $1.8 m$ (D) $1 m$
18. A sinusoidal voltage of amplitude $15V$ is connected the input terminals of the circuit shown in the figure. Assume that the diodes are ideal. In the output waveform:



- (A) Positive peaks of the input will be clipped to $+12V$ and negative peaks will be clipped to $-6V$
 (B) Positive peaks of the input will be clipped to $+6V$ and negative peaks will be clipped to $-12V$
 (C) Positive peaks of the input will be clipped to $+12V$ and negative peaks will be clipped to $-12V$
 (D) Positive peaks of the input will be clipped to $+6V$ and negative peaks will be clipped to $-6V$
19. Two different coils have self-inductances $I_1 = 8mH$, $I_2 = 2mH$. The current in both the coils is increased at the same constant rate. At a certain instant of time, the current, the induced voltage and energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then:
- (A) $i_1 / i_2 = \frac{1}{8}$ (B) $i_1 / i_2 = 8$ (C) $W_2 / W_1 = 4$ (D) $V_2 / V_1 = 4$
20. Two forces of equal magnitude act at a point making an angle θ with each other. If the direction of one of the forces is reversed, the direction of the resultant will turn through:
- (A) 30° (B) 45° (C) 60° (D) 90°

SPACE FOR ROUGH WORK

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

1. A body starts from rest and travels with uniform acceleration such that it covers 8 m during the 2nd second. During the 5th second it would travel a distance _____ (in m).
2. A particle is moving along a circular path with a constant speed of 10ms^{-1} . What is the magnitude of the change in velocity (m/s) of the particle, when it moves through an angle of 60° around the centre of the circle?
3. The electric potential (in volt) in a region along the x -axis varies with x according to the relation $V(x) = 5 + 4x^2$, where x is in m . Therefore, force experienced by a charge of 1C placed at $x = -1\text{m}$ is _____ N .
4. A 420m long rope of mass 5kg joins two rock climbers. One climber strikes the rope and the second one feels the effect 1.4s later. The tension in the rope is $\frac{n}{14}$ kilo newton. Find value of n _____.
5. A certain quantity of oxygen $\left(\gamma = \frac{7}{5}\right)$ is compressed isothermally until its pressure is doubled (p_2). The gas is then allowed to expand adiabatically until its original volume is restored. Then, the final pressure (p_3) in terms of initial pressure (p_1) is $p_3 = \frac{p_1}{4^{1/n}}$. The value of n is _____.

SPACE FOR ROUGH WORK

6. A particle of mass 0.2 kg moves along a path given by the relation $\vec{r} = 2 \cos \omega t \hat{i} + 3 \sin \omega t \hat{j}$ in meter. Then, the torque on the particle about the origin is $(n-3)$ in Nm . Find value of n _____.
7. A sinusoidal voltage of peak value 283V and angular frequency 320 rad/s is applied to a series LCR circuit. Given that $R = 5\Omega$, $L = \frac{1}{40} \text{ H}$ and $C = 1000 \mu\text{F}$. The phase difference between the voltage across the source and the current is nearly $\frac{\pi}{n}$ radian, where n is _____.
8. Ice at -20°C is added to 50 g of water at 40°C . When the temperature of the mixture reaches 0°C , it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to _____ g.
(Specific heat of water $= 4.2 \text{ J/g}^\circ\text{C}$ Specific heat of Ice $= 2.1 \text{ J/g}^\circ\text{C}$
Heat of fusion of water at $0^\circ\text{C} = 334 \text{ J/g}$)
9. A ball dropped on ground from a height of 1m rises to a height of 75cm on rebound. When thrown down from the same height with a velocity of 2m/s , it would rise to a height _____ cm.
(Assume $g = 10 \text{ m/s}^2$)
10. A myopic adult has a far point at 0.1m . His power of accommodation is 4 dioptres. What is his near point with glass in cm? (Take the image distance from the lens of the eye to the retina to be 2cm .)

SPACE FOR ROUGH WORK

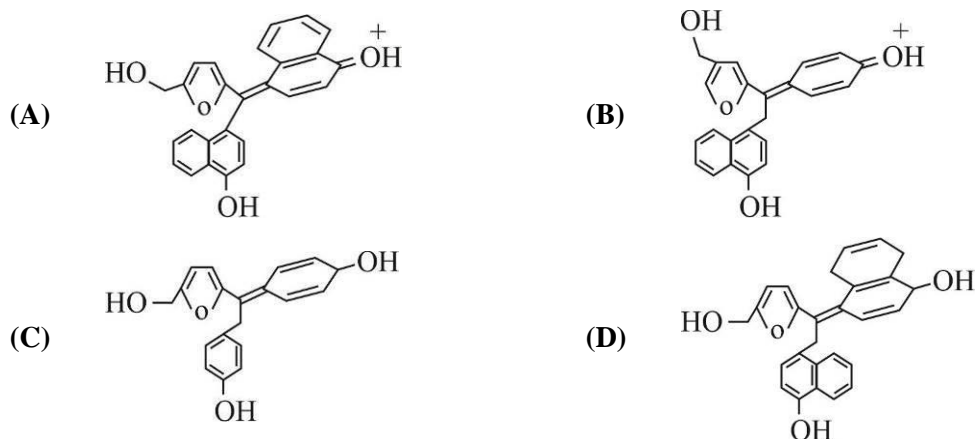
PART - II : CHEMISTRY**100 MARKS****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. The maximum number of possible oxidation states of actinoides are shown by:
(A) neptunium (Np) and plutonium (Pu) (B) actinium (Ac) and thorium (Th)
(C) nobelium (No) and lawrencium (Lr) (D) berkelium (Bk) and californium (Cf)
2. The pH of $0.02\text{M NH}_4\text{Cl}$ solution will be [given $K_b(\text{NH}_4\text{OH}) = 10^{-5}$ and $\log 2 = 0.301$]
(A) 4.65 (B) 5.35 (C) 2.65 (D) 4.35
3. The set in which compounds have different nature is:
(A) B_2O_3 and N_2O_3 (B) NO and CO
(C) Cr_2O_3 and Sc_2O_3 (D) CrO_3 and Mn_2O_7

SPACE FOR ROUGH WORK

4. The correct structure of redish-purple coloured complex formed in reaction of molisch's test for carbohydrates is:

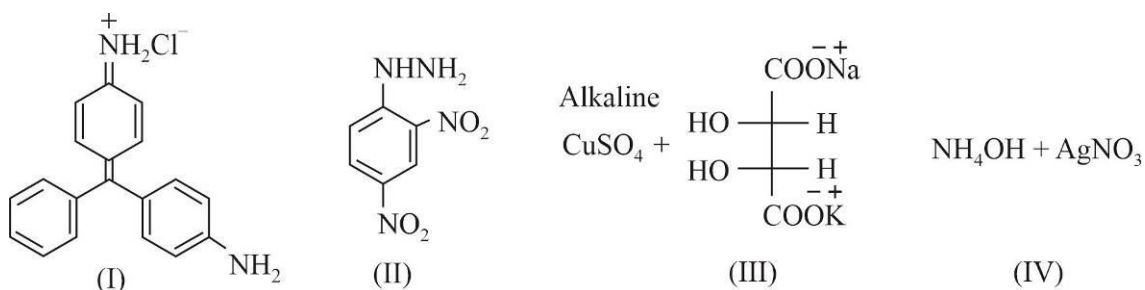


5. For the reaction of H_2 with I_2 , the equilibrium constant is $2.5 \times 10^{-4} \text{ dm}^3 \text{ mol}^{-1} \text{ s}^{-1}$ at 327°C and $1.0 \text{ dm}^3 \text{ mol}^{-1} \text{ s}^{-1}$ at 527°C . The activation energy for the reaction, in kJ mol^{-1} is:

($R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$)

- (A) 150 (B) 72 (C) 166 (D) 59

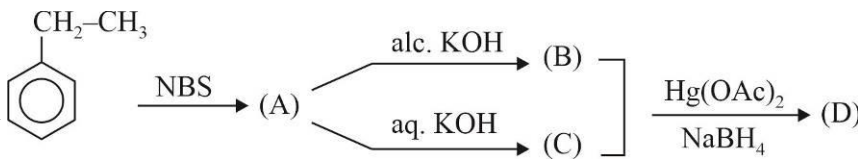
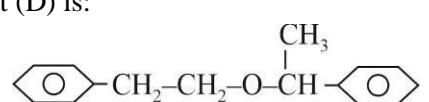
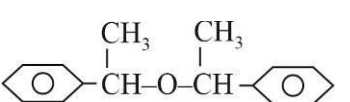
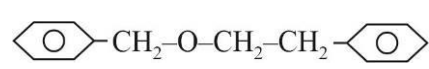
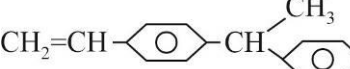
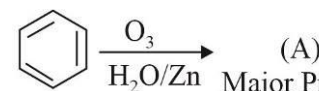
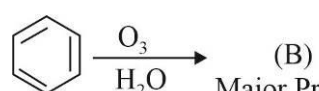
6.



Which among the above compounds can be used to distinguish between Benzaldehyde and acetophenone?

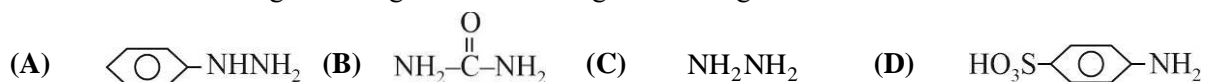
- (A) Only (II) (B) Only (IV)
(C) (III) and (IV) only (D) (I), (III) and (IV) only

SPACE FOR ROUGH WORK

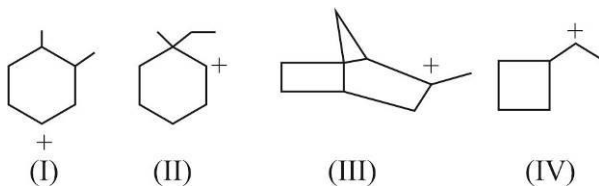
7. The crystal field stabilization energy (CFSE) of $[\text{Fe}(\text{H}_2\text{O})_6]\text{Cl}_2$ and $\text{K}_2[\text{NiCl}_4]$ respectively, are:
 (A) $-0.4\Delta_0$ and $-1.2\Delta_t$ (B) $-0.4\Delta_0$ and $-0.8\Delta_t$
 (C) $-2.4\Delta_0$ and $-1.2\Delta_t$ (D) $-0.6\Delta_0$ and $-0.8\Delta_t$
8. For the reaction,
 $2\text{SO}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2\text{SO}_3(\text{g})$,
 $\Delta H = -57.2 \text{ kJ mol}^{-1}$ and $K_c = 1.7 \times 10^{16}$.
 Which of the following statement is INCORRECT?
 (A) The equilibrium constant decreases as the temperature increases
 (B) The addition of inert gas at constant volume will not affect the equilibrium constant
 (C) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.
 (D) The equilibrium will shift in forward direction as the pressure increases.
9. The species given below that does NOT show disproportionation reaction is:
 (A) $\text{H}-\overset{\text{O}}{\parallel}{\text{C}}-\text{H}$ (B) HNO_2 (C) H_2PO_2^- (D) SO_4^{2-}
10. In the given reaction 
 Product (D) is:
 (A)  (B) 
 (C)  (D) 
11.  Major Product (A)
 Major Product (B)
 For above chemical reactions, identify the correct statement from the following.
 (A) Compound 'A' is Glyoxal and compound 'B' is oxalic acid
 (B) Both compound 'A' and compound 'B' are oxalic acid
 (C) Compound 'A' is oxalic acid and compound 'B' is Glyoxal
 (D) Both compound 'A' and compound 'B' are Glyoxal

SPACE FOR ROUGH WORK

12. Which of the following will not give test for nitrogen in lassaigne's test?



13.



Among the given carbocations rearrangement is favourable.

- (A) (I), (II) and (III) only (B) (II) and (IV) only
(C) (II) only (D) (I) and (II) only

14. The correct order of the wavelength of absorption for the given complex.

- (A) $[\text{CrCl}_6]^{3-} > [\text{Cr}(\text{H}_2\text{O})_6]^{3+} > [\text{Cr}(\text{NH}_3)_6]^{3+}$
(B) $[\text{Cr}(\text{H}_2\text{O})_6]^{3+} > [\text{CrCl}_6]^{3-} > [\text{Cr}(\text{NH}_3)_6]^{3+}$
(C) $[\text{Cr}(\text{NH}_3)_6]^{3+} > [\text{Cr}(\text{H}_2\text{O})_6]^{3+} > [\text{CrCl}_6]^{3-}$
(D) $[\text{CrCl}_6]^{3-} > [\text{Cr}(\text{NH}_3)_6]^{3+} > [\text{Cr}(\text{H}_2\text{O})_6]^{3+}$

15. According to the Crystal Field Theory, which one of the following compounds is an inner orbital complex?

- (A) $[\text{NiCl}_2(\text{PPh}_3)_2]$ (B) $[\text{Ni}(\text{CN})_2(\text{PPh}_3)_2]$
(C) $[\text{Co}(\text{H}_2\text{O})_3\text{F}_3]$ (D) $[\text{NiF}_6]^{4-}$

SPACE FOR ROUGH WORK

16. Identify the incorrect statement from the following:
(A) β -1, 4 glycosidic linkage makes cellulose polymer
(B) α -amylose contains unbranched chain
(C) All polysaccharides are reducing sugars
(D) In DNA, cytosine and guanine base pairs are bonded with three hydrogen bonding
17. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z respectively, are :
(A) K_2MnO_4 , KMnO_4 and Cl_2 (B) KMnO_4 , K_2MnO_4 and Cl_2
(C) K_2MnO_4 , KMnO_4 and SO_2 (D) K_3MnO_4 , K_2MnO_4 and Cl_2
18. The molar solubility of $\text{Cd}(\text{OH})_2$ is $1.84 \times 10^{-5} \text{ M}$ in water. The expected solubility of $\text{Cd}(\text{OH})_2$ in a buffer solution of pH = 12 is :
(A) $\frac{2.49}{1.84} \times 10^{-9} \text{ M}$ (B) $2.49 \times 10^{-10} \text{ M}$ (C) $6.23 \times 10^{-11} \text{ M}$ (D) $1.84 \times 10^{-9} \text{ M}$
19. A pale green crystalline metal salt of M dissolves freely in water. On standing it gives a brown ppt. on addition of aqueous NaOH. The metal salt solution also gives a black ppt. on bubbling H_2S in basic medium. An aqueous solution of the metal salt decolourises the pink colour of the permanganate solution. The metal in the metal salt solution is:
(A) Pb (B) Co (C) Fe (D) Hg
20. The INCORRECT match in the following is :
(A) $\Delta G^\circ > 0, K < 1$ (B) $\Delta G^\circ < 0, K < 1$
(C) $\Delta G^\circ = 0, K = 1$ (D) $\Delta G^\circ < 0, K > 1$

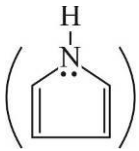
SPACE FOR ROUGH WORK

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

1. To synthesise 1.0 mole of product from ethyl-3-oxobutanoate _____ equivalents of CH_3MgBr reagent will be required. (Integer value)
2. 250mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of NaCl molecules in the solution after complete reaction is _____ $\times 10^{20}$. (Nearest integer) ($N_A = 6 \times 10^{23}$)
3. An average person if needs 100 kJ of muscular work to walk one kilometer then what is maximum distance (in km) that a person will able to walk after eating 160 gm of glucose _____. (Nearest integer) (Use : $\Delta_C H(\text{glucose}) = -2800 \text{ kJ mol}^{-1}$) [Molar mass of glucose = 180 g mol^{-1}]
4. The number of nitrogen atoms in a molecule produced when benzaldehyde is reacted with 2, 4-DNP.
5. A viral preparation was inactivated in a chemical bath. The inactivation process was found to be of first order in virus concentration, and at the beginning of the experiment 25% of the virus was found to be inactivated per minute. The rate constant for viral inactivation is _____ $\times 10^{-4} \text{ sec}^{-1}$. (Nearest integer) [Use: $\log 4 = 0.6$; $\log 3 = 0.48$]
6. At 25°C , the vapour pressure of CCl_4 is 115 mm Hg and that of SiCl_4 is 230 mm Hg. The mole fraction of CCl_4 in the vapour phase at 25°C if solution is made by mixing equal masses of CCl_4 and SiCl_4 is _____ $\times 10^{-3}$. (Nearest integer) (Mass number of Si = 28, Cl = 35.5, C = 12)

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7. The number of delocalisable π electrons in pyrrole  molecule is _____.
8. The total number of unpaired electrons in t_{2g} orbitals of central atom in $[\text{CoF}_6]^{3-}$ ion is _____.
[Atomic number of Co is 27]
9. At 1400 K, for the reaction in equilibrium:
- (i) $3\text{H}_{2(g)} + \text{SO}_{2(g)} \longrightarrow \text{H}_2\text{S}_{(g)} + 2\text{H}_2\text{O}_{(g)}$, $K_{p1} = 1.8 \times 10^8$
- (ii) $\text{H}_{2(g)} + \frac{1}{2}\text{S}_{2(g)} \longrightarrow \text{H}_2\text{S}_{(g)}$, $K_{p2} = 8 \times 10^{-2}$
- Thus for the equilibrium
- (iii) $4\text{H}_{2(g)} + 2\text{SO}_{2(g)} \longrightarrow \text{S}_{2(g)} + 4\text{H}_2\text{O}_{(g)}$, $K_p = 5 \times 10^x$. Value of x is _____.
10. Number of electrons with $(\ell + m_\ell) = 0$ for the calcium element is _____.
(Atomic number of Ca = 20)

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PART - III : MATHEMATICS**100 MARKS****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

- $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0, y_2 < 0$ be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of the parabolas with latus rectum PQ are:

(A) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$ (B) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$
 (C) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$ (D) Both B and C
- Let $g(x)$ be a continuous and differentiable function such that $\int_0^2 \left\{ \int_{\sqrt{x}}^{\sqrt{5/2}} [2x^2 - 3] dx \right\} \cdot g(x) dx = 0$, then $g(x) = 0$ when $x \in (0, 2)$ has: (Where $[.]$ denote greatest integer function)

(A) Exactly one real root (B) Atleast one real root
 (C) No real root (D) Two real roots
- The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12, 14 then absolute differences of remaining observations will be:

(A) 2 (B) 14 (C) 8 (D) 6
- Let $f(x)$ is a continuous function for all real values of x and satisfies $\int_0^x f(t) dt = \int_x^1 t^2 \cdot f(t) dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$ then value of 'a' is equal to:

(A) $-\frac{1}{24}$ (B) $\frac{17}{168}$ (C) $\frac{1}{7}$ (D) None of these
- If 3 lies between the roots of the equation $2x^2 - (a-1)x + 8 = 0$, then the least integral value of a is:

(A) 8 (B) 9 (C) 10 (D) 11

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6. Let $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$, then $\int_0^{\pi/2} f(x) dx$ is:
- (A) $-\left(\frac{\pi}{4} + \frac{8}{15}\right)$ (B) $+\left(\frac{\pi}{4} - \frac{8}{15}\right)$ (C) $-\left(\frac{\pi}{8} + \frac{4}{15}\right)$ (D) $+\left(\frac{\pi}{8} - \frac{4}{15}\right)$
7. The complex number $z = 1 + i$ is rotated through an angle $\frac{3\pi}{2}$ in anticlockwise direction about the origin and stretched by additional $\sqrt{2}$ unit, then the new complex number is:
- (A) $-\sqrt{2} - \sqrt{2}i$ (B) $\sqrt{2} - \sqrt{2}i$ (C) $2 - \sqrt{2}i$ (D) None of these
8. In a $\triangle AEX$, T is the mid-point of XE , and P is the mid-point of ET . If the $\triangle APE$ is equilateral of side length equal to unity then which of the following alternative is not correct?
- (A) $AX = \sqrt{13}$ (B) $\angle EAT = 90^\circ$
(C) $\cos \angle XAE = \frac{-1}{\sqrt{13}}$ (D) $AT = \frac{1}{\sqrt{3}}$
9. If the mapping $f(x) = ax + b$, $a < 0$ maps $[-1, 1]$ onto $[0, 2]$, then for all values of ' θ ', $A = \cos^2 \theta + \sin^4 \theta$ is such that:
- (A) $f(0) \leq A \leq f(-2)$ (B) $f\left(\frac{1}{4}\right) \leq A \leq f(0)$
(C) $f\left(\frac{1}{3}\right) \leq A \leq f(0)$ (D) $f(-1) \leq A \leq f(-2)$
10. If $y = y(x)$ and $\left(\frac{2 + \sin x}{y + 1}\right) \frac{dy}{dx} = -\cos x$, $y(0) = 1$, then $y\left(\frac{\pi}{2}\right) =$
- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) 1

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11. The coefficient of x^n in the expansion of $\frac{1}{(1-x)(3-x)}$ is:
- (A) $\frac{3^{n+1}-1}{2 \cdot 3^{n+1}}$ (B) $\frac{3^{n+1}-1}{3^{n+1}}$ (C) $2\left(\frac{3^{n+1}-1}{3^{n+1}}\right)$ (D) None of these
12. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2+bx+c=0$ and $px^2+qx+r=0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0, \beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then $\frac{b^2}{q^2} =$
- (A) $\frac{ac}{pr}$ (B) $\frac{ar}{pc}$ (C) $\frac{ap}{cr}$ (D) None of these
13. Let F_1 and F_2 be the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and point $B(0, b)$ be an end point of the minor axis. If triangle BF_1F_2 is equilateral, e is the eccentricity of the ellipse and Δ is the area of the triangle BF_1F_2 , then:
- (A) $e = \frac{1}{4}$ (B) $e = \frac{1}{3}$ (C) $\Delta = \frac{\sqrt{3}}{4}a^2$ (D) $\Delta = \sqrt{3}a^2$
14. Minimum value of $n \in N$ such that $\cot^{-1}\left(\frac{n^2-10n+19}{\sqrt{3}}\right) > \frac{\pi}{6}$ is:
- (A) 4 (B) 3 (C) 5 (D) 6
15. A particle starts at origin and moves along the x -axis in such a way that its velocity at point $(x, 0)$ is given by $\frac{dx}{dt} = \cos^2 \pi x$. Then particle never reaches point, on:
- (A) $x = \frac{1}{4}$ (B) $x = \frac{3}{4}$ (C) $x = \frac{1}{2}$ (D) $x = 1$

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16. The function f , defined by $f(x) = \frac{x^2}{2} + \ln|x| - 2\cos x$ increases for $x \in$
- (A) R^- (B) R^+ (C) $R - \{0\}$ (D) $[1, \infty)$
17. If $f(x)$ is a continuous function $\forall x \in R$ and the range of $f(x) = (2, \sqrt{26})$ and $g(x) = \left[\frac{f(x)}{a} \right]$ is continuous $\forall x \in R$ ($[.]$ denotes the greatest integer function), then the least positive integral value of a is:
- (A) 2 (B) 3 (C) 6 (D) 5
18. Total number of 3 letters words that can be formed from the letters of the word 'SAHARANPUR', is equal to:
- (A) 210 (B) 237 (C) 247 (D) 227
19. A five-digit number (having all different digits) is formed using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. The probability that the formed number either begins or ends with an odd digit, is equal to:
- (A) $\frac{5}{6}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$
20. The point $(-2m, m+1)$ is an interior point of the smaller region bounded by circle $x^2 + y^2 = 4$ and the parabola $y^2 = 4x$ then:
- (A) $-1 < m < -5 + 2\sqrt{6}$ (B) $-1 < m < \frac{3}{5}$
- (C) $0 < m < 4$ (D) $-5 - 2\sqrt{6} < m < 1$

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SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

- Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} + \hat{j}$, $\vec{d} = 5\hat{i} + 2\hat{j} + \hat{k}$. Consider the sequence of vectors $\vec{x}_1 = \vec{a}$, $\vec{x}_2 = \vec{c} + \left(\frac{(\vec{x}_1 - \vec{c}) \cdot \vec{d}}{|\vec{d}|^2} \right) \vec{d}$, $\vec{x}_3 = \vec{a} + \left(\frac{(\vec{x}_2 - \vec{a}) \cdot \vec{b}}{|\vec{b}|^2} \right) \vec{b}$, $\vec{x}_4 = \vec{c} + \left(\frac{(\vec{x}_3 - \vec{c}) \cdot \vec{d}}{|\vec{d}|^2} \right) \vec{d}$, $\vec{x}_5 = \vec{a} + \left(\frac{(\vec{x}_4 - \vec{a}) \cdot \vec{b}}{|\vec{b}|^2} \right) \vec{b}$, $\vec{x}_6 = \vec{c} + \left(\frac{(\vec{x}_5 - \vec{c}) \cdot \vec{d}}{|\vec{d}|^2} \right) \vec{d}$ and so on $\lim_{n \rightarrow \infty} |\vec{x}_{2n} - \vec{x}_{2n+1}| =$
- If the matrix $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ is orthogonal, then the value of $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}$ is equal to _____.
- Non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a} \cdot \vec{b} = 0$, $(\vec{b} - \vec{a}) \cdot (\vec{b} + \vec{c}) = 0$ and $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$ then the value of μ is _____.
- If the binomial expansion of $(a + bx)^{-2}$ is $\frac{1}{4} - 3x + \dots$, where $a > 0$, then $a + b$ is _____.
- Let \vec{a}, \vec{b} and \vec{c} be three vectors such that $|\vec{a}| = 2|\vec{b}| = 4|\vec{c}|$ and $2\vec{a} - 3\vec{b} + 6\vec{c} = \vec{0}$. If θ is the angle between \vec{a} and \vec{b} , then $3\cos\theta$ is equal to _____.

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6. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R . If eccentricity of the ellipse E_2 is e then $6e$ is equal to _____.
7. Let $\alpha, \beta (\alpha < \beta)$ be the solutions of $\sum_{n=1}^6 \frac{1}{\sin\left(\theta + (n-1)\frac{\pi}{4}\right)\sin\left(\theta + \frac{n\pi}{4}\right)} = 4\sqrt{2}$ and determinant value of A where $A = \begin{pmatrix} \sin \beta & \sin \alpha \\ \sin \alpha & \sin \beta \end{pmatrix}$ is k then $8k^2 =$ _____.
8. The total number of three-digit numbers, the sum of whose digits is even, is equal to _____.
9. The perpendicular distance of the point whose position vector is $(1, 3, 5)$ from the line $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ is equal to: _____.
10. $\lim_{x \rightarrow 2} \{[2-x] + [x-2] - x\}$ is equal to k then $|k|$ _____.

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••• End of JEE Main Home Practice Test - 2 [JEE - 2024] •••